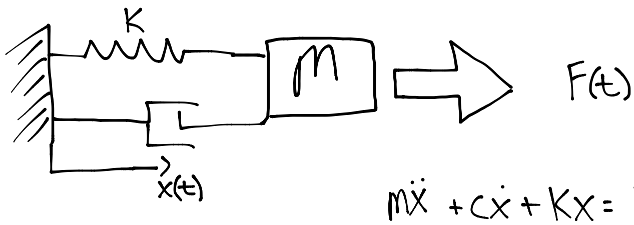
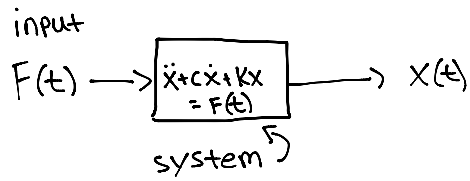
Input-output linear systems

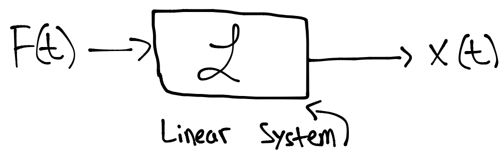


$m\ddot{x} + c\dot{x} + Kx = F(t)$ → no causality, just an equation

One view:



Linear System:



If $F_1(t) = X_1(t)$

$F_2(t) = X_2(t)$

Then $C_1F_1 + C_2F_2 = C_1X_1 + C_2X_2$

true for all F_1 and F_2

"Linear combination of known inputs of corresponding outputs"

- output is some linear combination

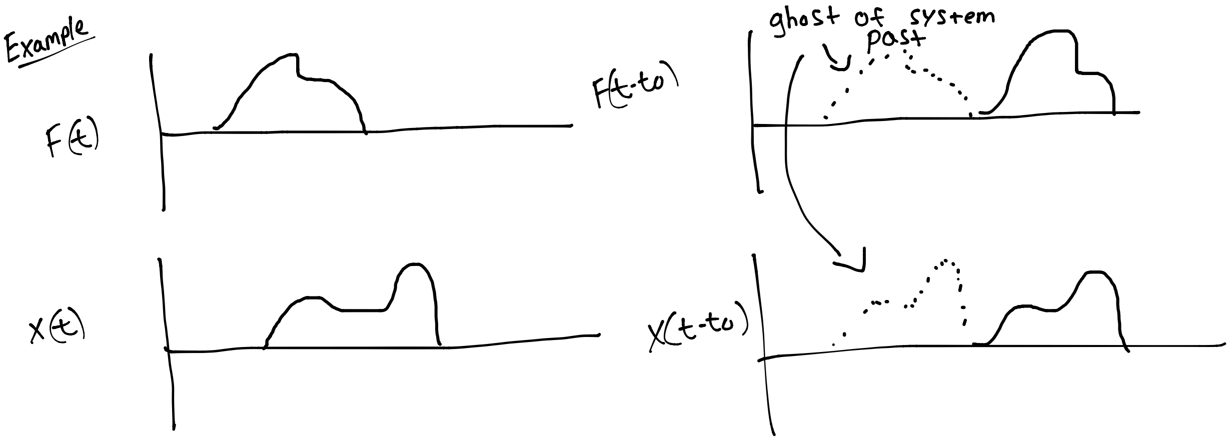
"Time invariant" = "autonomous"

└ not like in autonomous robots

"Time independent"

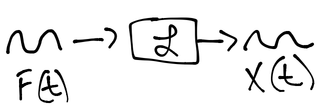
- for all $F(t)$ in, you get $X(t)$ out
  - $F(t - t_0) = X(t - t_0)$

Example



Common set of engineering systems are: Linear
Stable
Time Invariant

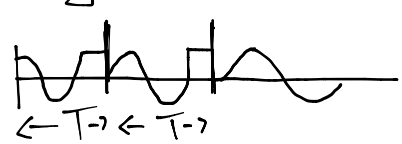
1 amazing property: Sine wave in → sine wave out



- could be different phase, different amplitude, but same frequency and shape
- get the idea of using

Fourier Series

Any periodic function $F(t) = A_0 + \sum A_n \cos \omega_n t + B_n \sin \omega_n t$

$F(t) =$ 

- You can write any function as the sum of sine waves

- already know if you put $\cos \omega_n t$ in, you get $X^c_{cn} \cos \omega_n t + X^c_{sn} \sin \omega_n t$

  $\sin \omega_n t$ in, you get " X^s_{cn} X^s_{sn} "

  frequency response

$$F(t) = A_0 + \sum A_n \cos W_n t + B_n \sin W_n t$$

$$\text{output} = A_0 F_0 + \sum A_n \left[X^c_{cn} \cos W_n t + X^c_{sn} \sin W_n t \right] + \sum B_n \left[X^s_{cn} \cos W_n t + X^s_{sn} \sin W t \right]$$

Sin response follows from the cosine response

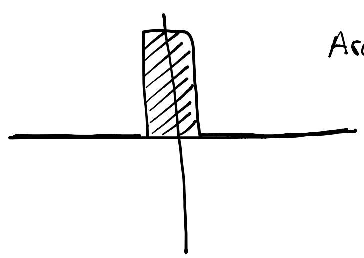
Same ideas apply with Fourier Transform (in Tongue)

= sum of δ functions

math: technique for solving diff eqs

engineering: only have to input sin waves, and we know all outputs/responses

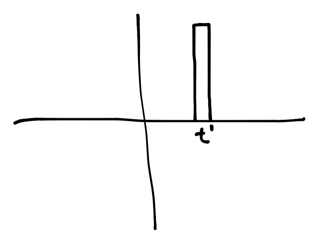
$\delta(t)$



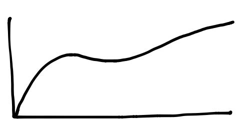
Area under the car is 1, even though highness goes to infinity and narrowness goes to zero

$\delta(t - t')$

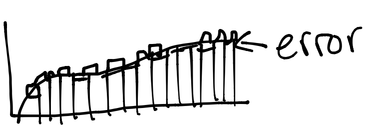
↑ constant



$F(t)$



$\approx$



$\sum \delta$ -> sum of delta functions

Reimann Sum

as integral

$$F(t) = \int_{-\infty}^{\infty} F(t')\, \delta(t - t')\, dt'$$

$$\delta(t) \longrightarrow \boxed{\mathcal{L}} \longrightarrow h(t)$$

$$\delta(t-t') \longrightarrow \boxed{\mathcal{L}} \longrightarrow h(t-t')$$

Convolution Integral

$$X(t) = \int_{-\infty}^{\infty} F(t')\,h(t-t')\,dt'$$

- any function is a sum of δ functions
- any response is a sum of δ responses